Minijet Initial Conditions For Non-Equilibrium Parton Evolution at RHIC and LHC

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Abstract

An important ingredient for the non-equilibrium evolution of partons at RHIC and LHC is to have some physically reasonable initial conditions for the single particle phase space distribution functions for the partons. We consider several plausible parametrizations of initial conditions for the single particle distribution function $f_i(x,p)$ and fix the parameters by matching $\int f(x,p)p^\mu d\sigma_\mu$ to the invariant momentum space semi-hard parton distributions obtained using perturbative QCD (pQCD), as well as fitting low order moments of the distribution function. We consider parametrizations of $f_i(x,p)$ with both boost invariant and boost non-invariant assumptions. We determine the initial number density, energy density and the corresponding (effective) temperature of the minijet plasma at RHIC and LHC energies. For a boost non-invariant minijet phase-space distribution function we obtain $\sim 30(140)$ /fm^3 as the initial number density, $\sim 50(520)$ GeV/fm^3 as the initial energy density and $\sim 520(930)$ MeV as the corresponding initial effective temperature at RHIC(LHC).

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In order to study thermalization of quark-gluon plasma at RHIC and LHC in any kinetic approach, one needs suitable initial conditions for the single particle phase-space distribution of partons. The hard and semi hard parton production can be calculated by using perturbative QCD (pQCD) [1]. The soft parton production cannot be calculated in a kinetic approach within perturbative QCD. One approximate strategy for looking at soft parton production is to assume the creation of a classical chromofield [2, 3, 4]. The first problem one faces in specifying initial conditions for the phase space single particle distribution function $f(x, p, t_0)$ of partons is that from pQCD one has information only about the momentum space distribution of minijets. The pQCD calculation (using scattering matrix element squared and the structure functions) is done in an approach which calculates probabilities between initial and final states so that any information about space-time evolution is lost. Thus we need to supplement pQCD with assumptions about the dependence of $f(x, p)$ on x.

In this paper our strategy will be to use pQCD to determine the single particle distribution function for the partons in momentum space, and then by taking simple parametric forms for $f(x, p)$ constrain the parametrization by fitting the single particle distribution as well as the average transverse momentum. This process can be refined by adding more parameters and fitting further moments of the jet distribution function. Of course, one hopes that the details of the parametrization of the initial conditions do not significantly effect the thermalization process.

The lowest order pQCD inclusive ($2 \to 2$) minijet cross section per nucleon in A-A collision is given by:

$$\sigma_{jet} = \int dp_T dy_1 dy_2 \frac{2\pi p_T}{\hat{s}} \sum_{ijkl} x_1 \; f_{i/A}(x_1, p_T^2) \; x_2 \; f_{j/A}(x_2, p_T^2) \; \hat{\sigma}_{ij\to kl}(\hat{s}, \hat{t}, \hat{u}), \tag{1}$$

where $\hat{\sigma}_{ij\to kl}$ is the elementary pQCD parton cross sections for the process $ij \to kl$. As gluons are the dominant part of the total minijet production, we will only consider the processes $gq(\bar{q}) \to gq(\bar{q})$ and $gg \to gg$ in this paper. The partonic level cross sections for these processes are given by:

$$\hat{\sigma}_{gq\to gq} = \frac{\alpha_s^2}{\hat{s}}(\hat{s}^2 + \hat{u}^2)[\frac{1}{\hat{t}^2} - \frac{4}{9\hat{s}\hat{u}}], \tag{2}$$

$$\hat{\sigma}_{gg\to gg} = \frac{9\alpha_s^2}{2\hat{s}}[3 - \frac{\hat{u}\hat{t}}{\hat{s}^2} - \frac{\hat{u}\hat{s}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}]. \tag{3}$$

The rapidities y_1, y_2 and the momentum fractions x_1, x_2 are related by,

$$x_1 = p_T \; (e^{y_1} + e^{y_2})/\sqrt{s}, \qquad x_2 = p_T \; (e^{-y_1} + e^{-y_2})/\sqrt{s}, \tag{4}$$

with the kinematic relations:

$$\hat{s} = x_1 x_2 s, \qquad \text{and} \qquad \hat{t} = -\frac{\hat{s}}{2}[1 - \tanh(\frac{y_1 - y_2}{2})]. \tag{5}$$

To compute the minijet cross section using Eq. (1) we need to specify the minimum transverse momentum cut-off p_0 above which the incoherent parton picture is applicable. The momentum cut-off is $\sim$ 1 GeV at RHIC and $\sim$ 2 GeV at LHC, which are obtained from the argument that there is saturation of the gluon structure function at low x [5]. For our quantitative calculation we use p_0= 1 GeV at RHIC and 2 GeV at LHC. The nuclear modified parton distribution functions which includes the shadowing effects are given by $f_{i/A}(x,Q^2) = R_A(x,Q^2) f_{i/N}(x,Q^2)$ where A and N stand for nucleus and free nucleon respectively. In this paper we use the GRV98 set of parton distributions for the free nucleon distribution function $f_{i/N}(x,Q^2)$ [6] (in momentum space). The GRV98 analysis uses the low x HERA data on deep inelatstic scatterings along with data from other hard scattering processes at fixed Q. The Q^2 evolution of the parton distribution function is performed by using perturbative QCD evolution equations. We use the EKS98 numerical parametrizations for the ratio function $R_A(x,Q^2)$ [7]. The EKS98 parametrization uses NMC and E665 structure functions from deep inelastic lepton-nucleus collisions and E772 Drell-Yan data from proton-nucleus collisions at fixed Q. The Q^2 evolution of the structure function is studied by using the DGLAP evolution equation. We multiply the above minijet cross section by a standard K factor (K=2), to account for the higher order contributions.

For central collisions, the minijet cross section (Eq. (1)) can be related to the total number of partons (N^{jet}) by

$$\frac{dN^{jet}}{dy dp_T} = K\ T(0) \int dy_2 \frac{2\pi p_T}{\hat{s}} \sum_{ijkl} x_1\ f_{i/A}(x_1,p_T^2)\ x_2\ f_{j/A}(x_2,p_T^2)\ \hat{\sigma}_{ij\to kl}(\hat{s},\hat{t},\hat{u}), \tag{6}$$

where $T(0) = 9A^2/8\pi R_A^2$ fm is the nuclear geometrical factor for head-on AA collisions (for a nucleus with a sharp surface). Here $R_A = 1.1A^{1/3}$ is the nuclear radius. Similarly the transverse energy distribution of the minijets are given by:

$$\frac{dE_T^{jet}}{dy dp_T} = K\ T(0) \int dy_2 \frac{2\pi p_T^2}{\hat{s}} \sum_{ijkl} x_1\ f_{i/A}(x_1,p_T^2)\ x_2\ f_{j/A}(x_2,p_T^2)\ \hat{\sigma}_{ij\to kl}(\hat{s},\hat{t},\hat{u}). \tag{7}$$

Our numerical results for the p_T distribution of the minijets computed from eq. (6) are plotted in Fig. 1.

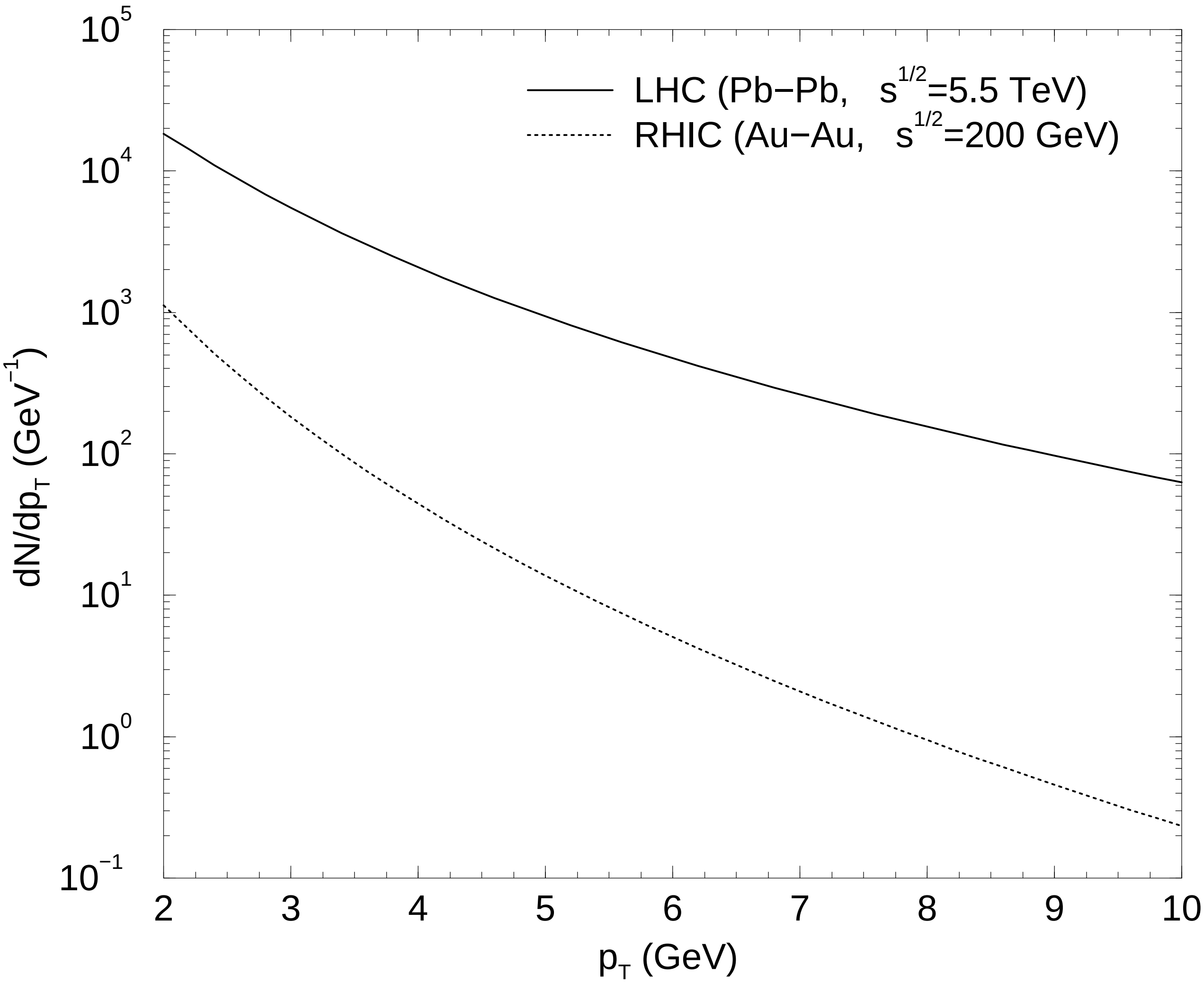

FIG. 1: Transverse momentum distribution of the initial minijets at RHIC and LHC

The rapidity distributions of the minijet production at RHIC and LHC are obtained by integrating over p_T for $p_T > p_0$ are shown in Fig. 2. It can be seen that the minijet distribution is not flat over the whole rapidity range. Hence it is not a particularly good approximation to assume boost invariance [8] and restrict oneself to the mid rapidity region. One important question that the transport equations should answer is how different the final rapidity distribution of the hadrons (or other signatures) from the rapidity distribution of the minijets at the pre-equilibrium stage are [9]. Of course this needs to be supplemented by further dynamical evolution near the hadronic phase transition. For this reason it is not appropriate to take into account just the flat minijet rapidity distribution seen in the mid rapidity region when evolving the minijet plasma. Partons formed outside the mid

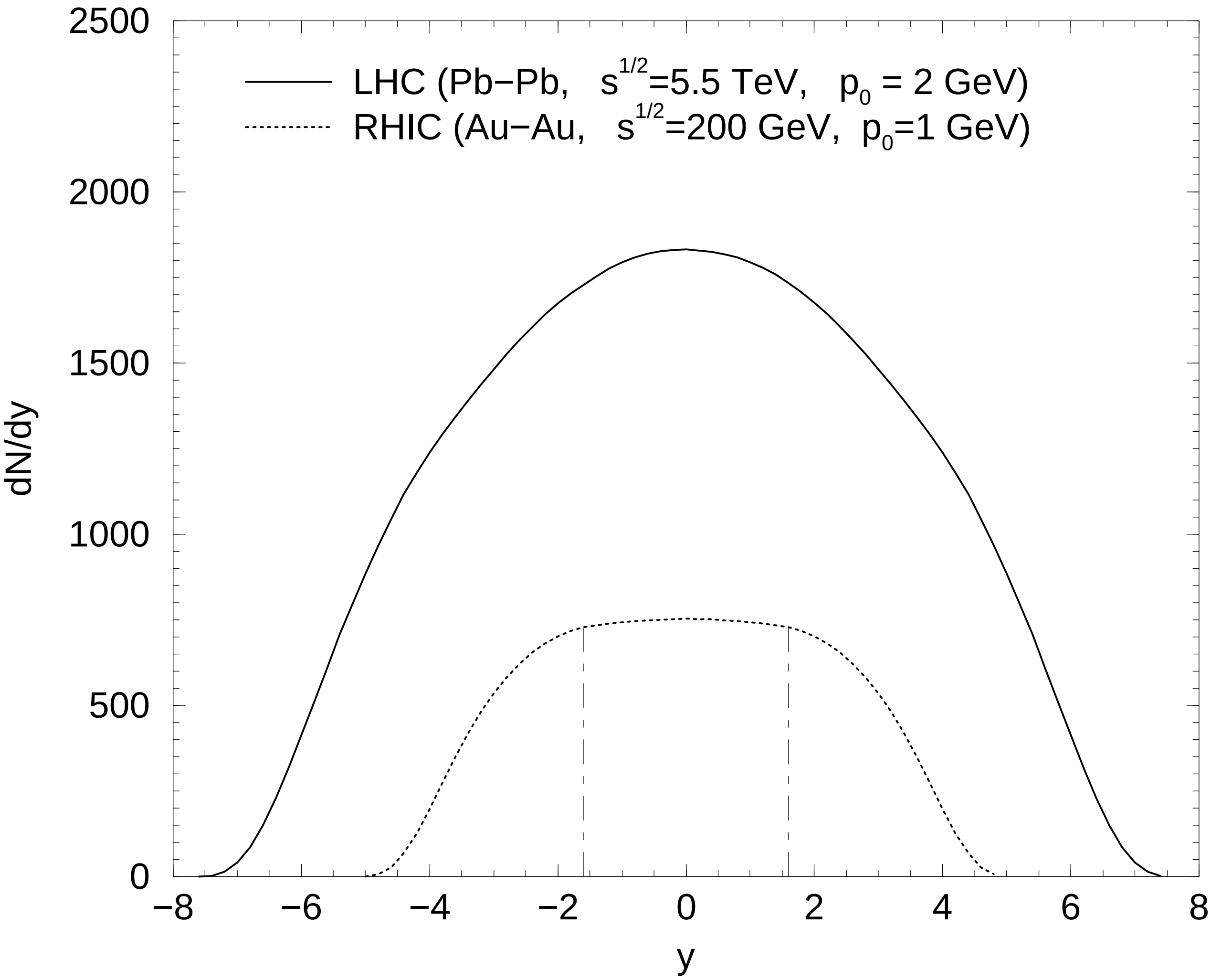

FIG. 2: Rapidity distribution of the initial minijets at RHIC and LHC

rapidity region do propagate in the pre-equilibrium stage with their rescatterings and hence one must consider all the partons in determining the initial condition of the QGP in the pre-equilibrium stage.

Thus we require additional information about how these partons are distributed in coordinate space. A simple ansatz based on scaling ([8]) allows extracting energy density from the transverse energy E_T distribution via:

$$\epsilon = \frac{1}{\pi R_A^2 \tau_0} \frac{dE_T}{dy} \tag{8}$$

This formula is correct if partons are uniformly distributed in coordinate space and the momentum rapidity y is equal to the coordinate rapidity η. For an 1+1 dimensional expanding system coordinate rapidity η is defined via: $t = \tau \cosh \eta$ and $z = \tau \sinh \eta$.

Traditionally the above formula is widely applied to obtain energy density from E_T distribution of minijets. Although the above formula might be applicable in a situation where the minijet distribution is flat in rapidity, it is precisely the scale breaking effects that we hope to understand from our transport approach. As it can be seen from Fig. 2 the initial minijet distribution function is not flat in the entire rapidity range and hence the above formula is not applicable in the pre-equilibrium region. Also it is not clear that partons are uniformly distributed at the initial time and just dividing by a volume (see eq. (8)) to obtain the energy density from the total energy is a rather crude estimate of initial conditions. To get a better estimate for the initial conditions one should take into account the correlation between coordinate rapidity η and momentum rapidity y. We will assume this correlation has a particular form, and then look at the sensitivity of the global quantities to the parametrization.

For an expanding system the minijet number distribution eq. (6) can be related to the phase-space distribution function via [10]:

$$\frac{d^3N^{jet}}{\pi dy dp_T^2} = g_C \int f(x,p)\ p^\mu d\sigma_\mu. \tag{9}$$

where $g_C = 16$ is the product of spin and color degrees of freedom,

$$d\sigma^\mu = \pi R_A^2 \tau d\eta(\cosh\eta, 0, 0, \sinh\eta), \tag{10}$$

and

$$p^\mu = (p_T\cosh y, p_T\cos\phi, p_T\sin\phi, p_T\sinh y), \tag{11}$$

for an 1+1 dimension expanding system. Using the above relations we get at the initial time τ_0 (=$1/p_0$):

$$\frac{dN^{jet}}{\pi dy dp_T^2} = g_C \pi R_A^2 \tau_0 \int d\eta\ p_T\cosh(\eta - y)\ f(p_T, \eta, y, \tau_0), \tag{12}$$

where we assume a transverse isotropy at the early stage. In the following we will obtain the initial phase-space distribution function of the gluon minijets by using both boost invariant and boost non-invariant schemes.

A. Boost Invariant Initial Distribution Function From Minijets

First let us assume that a boost invariant description [8] is appropriate. Then the gluon distribution function depends only on the boost invariant parameters τ, p_T and $\xi = \eta - y$.

We will parametrize the $\eta - y$ correlation as Gaussian:

$$f(p_T, \eta - y, \tau_0) = f(p_T) e^{-\frac{(\eta-y)^2}{\sigma^2(p_T)}}, \tag{13}$$

where $f(p_T)$ will be determined from eq. (12) by using eq. (6) which is obtained by using pQCD. Using the above equation in eq. (12) we get:

$$\frac{dN^{jet}}{dp_T} = g_C 2\pi^2 R_A^2 \tau_0 \; p_T^2 f(p_T) \int dy \int_{-\infty}^{\infty} d\eta \cosh(\eta - y) e^{-\frac{(\eta-y)^2}{\sigma^2(p_T)}} \tag{14}$$

which gives:

$$\frac{dN^{jet}}{dp_T} = g_C 2\pi^2 \sqrt{\pi} R_A^2 \tau_0 p_T^2 f(p_T) \sigma(p_T) e^{\sigma^2(p_T)/4} \int dy \tag{15}$$

where the maximum allowed value of the momentum rapidity at RHIC and LHC depends on p_T and $\sqrt{s}$. Integrating over y we get from the above equation:

$$f(p_T) = \frac{\frac{dN^{jet}}{dp_T}}{g_C 2\pi^2 \sqrt{\pi} R_A^2 \tau_0 p_T^2 2 \ln(\sqrt{s}/2p_T + \sqrt{s/4p_T^2 - 1})} \frac{e^{-\sigma^2(p_T)/4}}{\sigma(p_T)}, \tag{16}$$

which gives a boost invariant initial distribution function:

$$f(p_T, \xi, \tau_0) = \frac{\frac{dN^{jet}}{dp_T}}{g_C 2\pi^2 \sqrt{\pi} R_A^2 \tau_0 p_T^2 2 \ln(\sqrt{s}/2p_T + \sqrt{s/4p_T^2 - 1})} \frac{e^{-\frac{\xi^2}{\sigma^2(p_T)}}}{\sigma(p_T) e^{\sigma^2(p_T)/4}}. \tag{17}$$

Using this boost invariant initial phase-space distribution function of the gluon the initial minijet number density is given by:

$$\begin{aligned} n(\tau_0) &= g_C \int d\Gamma p^\mu u_\mu f(p_T, \eta - y, \tau_0) = g_C \int d^2p_T p_T \int d\xi \cosh\xi f(p_T, \xi, \tau) \\ &= 2\pi \int dp_T \frac{\frac{dN^{jet}}{dp_T}}{2\pi^2 R_A^2 \tau_0 2 \ln(\sqrt{s}/2p_T + \sqrt{s/4p_T^2 - 1})}, \end{aligned} \tag{18}$$

where $d\Gamma = \frac{d^3p}{p^0}$. Similarly the initial energy density is given by:

$$\begin{aligned} \epsilon(\tau_0) &= g_C \int d\Gamma (p^\mu u_\mu)^2 f(p_T, \eta - y, \tau_0) = g_C \int d^2p_T p_T^2 \int d\xi \cosh^2\xi f(p_T, \xi, \tau) \\ &= 2\pi \int dp_T p_T \frac{\frac{dN^{jet}}{dp_T}}{4\pi^2 R_A^2 \tau_0 2 \ln(\sqrt{s}/2p_T + \sqrt{s/4p_T^2 - 1})} (e^{-\sigma^2(p_T)/4} + e^{3\sigma^2(p_T)/4}). \end{aligned} \tag{19}$$

B. Boost Non-Invariant Initial Distribution Function From Minijets

In the above we used a boost invariant distribution function $f(\tau, \xi, p_T)$ as the initial phase-space distribution function. However this is not consistent with the actual rapidity distribution function obtained from pQCD. To relax the boost invariant assumption, we will introduce a function $f(p_T, y)$ instead of $f(p_T)$ in eq. (13). The form of the boost non-invariant distribution we take is:

$$f(p_T, \eta, y, \tau_0) = f(p_T, y) e^{-\frac{(\eta-y)^2}{\sigma^2(p_T)}}, \tag{20}$$

where $f(p_T, y)$ is obtained from $\frac{dN^{jet}}{dy dp_T}$. Note that $f(p_T)$ in eq. (13) is obtained from $\frac{dN^{jet}}{dp_T dy}$ after integrating over y. However, in the above form (eq. (20)), the y dependence is more general which for small σ^2 is close to that obtained by using pQCD. Using the above form in eq. (12) we get:

$$\frac{dN^{jet}}{dy dp_T} = g_C 2\pi^2 R_A^2 \tau_0 \; p_T^2 f(p_T, y) \int_{-\infty}^{\infty} d\eta \cosh(\eta - y) e^{-\frac{(\eta-y)^2}{\sigma^2(p_T)}} \tag{21}$$

which gives:

$$\frac{dN^{jet}}{dy dp_T} = g_C 2\pi^2 \sqrt{\pi} R_A^2 \tau_0 p_T^2 f(p_T, y) \sigma(p_T) e^{\sigma^2(p_T)/4}. \tag{22}$$

From the above equation we get:

$$f(p_T, y) = \frac{\frac{dN^{jet}}{dy dp_T}}{g_C 2\pi^2 \sqrt{\pi} R_A^2 \tau_0 p_T^2 \sigma(p_T) e^{\sigma^2(p_T)/4}} \tag{23}$$

which gives a boost non-invariant initial phase-space gluon distribution function:

$$f(p_T, \eta, y, \tau_0) = \frac{\frac{dN^{jet}}{dy dp_T}}{g_C 2\pi^2 \sqrt{\pi} R_A^2 \tau_0 p_T^2} \frac{e^{-\frac{(\eta-y)^2}{\sigma^2(p_T)}}}{\sigma(p_T) e^{\sigma^2(p_T)/4}}. \tag{24}$$

Using the above boost non-invariant gluon distribution function the initial minijet number density is given by:

$$\begin{aligned} n(\tau_0, \eta) &= g_C \int d\Gamma p^\mu u_\mu f(p_T, \eta, y, \tau_0) = g_C \int d^2p_T p_T \int dy \cosh(\eta - y) f(p_T, \eta, y, \tau_0) \\ &= 2\pi \int dp_T \int dy \frac{\frac{dN^{jet}}{dy dp_T}}{2\pi^2 \sqrt{\pi} R_A^2 \tau_0} \frac{e^{-\frac{(\eta-y)^2}{\sigma^2(p_T)}}}{\sigma(p_T) e^{\sigma^2(p_T)/4}} \cosh(\eta - y), \end{aligned} \tag{25}$$

and the initial minijet energy density given by:

$$\begin{aligned} \epsilon(\tau_0, \eta) &= g_C \int d\Gamma (p^\mu u_\mu)^2 f(p_T, \eta, y, \tau_0) = g_C \int d^2p_T p_T^2 \int dy \cosh^2(\eta - y) f(p_T, \eta, y, \tau_0) \\ &= 2\pi \int dp_T p_T \int dy \frac{\frac{dN^{jet}}{dy dp_T}}{2\pi^2 \sqrt{\pi} R_A^2 \tau_0} \frac{e^{-\frac{(\eta-y)^2}{\sigma^2(p_T)}}}{\sigma(p_T) e^{\sigma^2(p_T)/4}} \cosh^2(\eta - y). \end{aligned} \tag{26}$$

C. Initial Energy Density and Number Density at RHIC and LHC

Using the above expressions for the minijet initial phase-space distribution functions we can now predict the initial energy density and number density of the minijet plasma at RHIC and LHC. As temperature can not be defined in a non-equilibrium situation we define an *effective* temperature by:

$$T_{eff} = [\frac{15}{8\pi^2}\epsilon]^{1/4}, \tag{27}$$

with the non-equilibrium energy density obtained above.

Before we calculate the initial conditions we would like to find out the values of the unknown parameter σ^2 appearing in the above equation. We will fix this unknown parameter by equating the first p_T moment of the distribution function with the pQCD predicted E_T distribution as given by eq. (7). We have for the boost invariant case:

$$\int dp_T \frac{dE_T^{jet}}{dp_T} = 2g_C\pi^2 R_A^2\tau_0 \int dp_T\ p_T^3 f(p_T) = \int dp_T \frac{p_T \frac{dN^{jet}}{dp_T}}{\sqrt{\pi}2\ln(\sqrt{s}/2p_T + \sqrt{s/4p_T^2 - 1})\sigma(p_T)e^{\sigma^2(p_T)/4}} \tag{28}$$

and for the boost non-invariant case:

$$\int dp_T \int dy \frac{dE_T^{jet}}{dydp_T} = 2g_C\pi^2 R_A^2\tau_0 \int dp_T \int dy\ p_T^3 f(p_T, y) = \int dp_T \int dy \frac{p_T \frac{dN^{jet}}{dydp_T}}{\sqrt{\pi}\sigma(p_T)e^{\sigma^2(p_T)/4}} \tag{29}$$

We determine the value of σ^2 by using the pQCD values of $\frac{dE_T^{jet}}{dydp_T}$ and $\frac{dE_T^{jet}}{dp_T}$ from eq. (7) in the left hand side of the above equations. Assuming σ^2 independent of p_T we find from the above equations: $\sigma^2 = 0.0034(0.0015)$ at RHIC(LHC) for boost invariant case and $\sigma^2 = 0.28(0.28)$ at RHIC(LHC) for boost non-invariant case. Note that for boost non-invariant case σ^2 is same at RHIC and LHC as the normalization factor $\sqrt{\pi}\sigma(p_T)e^{\sigma^2(p_T)/4}$ in eq. (29) is energy independent whereas the corresponding normalization factor $\sqrt{\pi}2\ln(\sqrt{s}/2p_T + \sqrt{s/4p_T^2 - 1})\sigma(p_T)e^{\sigma^2(p_T)/4}$ in eq. (28) depends on energy. Using the above values for the boost invariant case in eq. (19) we find ϵ_0=31 GeV/fm^3 with T_{eff}= 460 MeV at RHIC and ϵ_0=287 GeV/fm^3 with T_{eff}= 800 MeV at LHC. The number density for the boost invariant case obtained from eq. (18) is n_0=20/fm^3 at RHIC and 85/fm^3 at LHC. Using the values of the σ^2 for boost non-invariant case we find from eq. (26): ϵ_0=51 GeV/fm^3 with T_{eff}=521 MeV at RHIC and ϵ_0=517 GeV/fm^3 with T_{eff}=932 MeV at LHC for η=0. The number density for the boost non-invariant case obtained from eq. (25) is n_0=30/fm^3 at RHIC and 138/fm^3 at LHC for η=0. It can be noted that the energy density for the boost invariant case is less than that of the boost non-invariant case. This is because we have assumed a boost invariance in the entire rapidity range. However, the actual pQCD minijet rapidity distribution is not flat in the entire rapidity range as can be seen from Fig. 2. Therefore, in order to maintain the boost invariance in the entire rapidity range the density has to be smaller. However, in boost non-invariant case the rapidity distribution of the minijet is minimaly altered and the pQCD form of $\frac{dN}{dp_T dy}$ is used throughout the calculation.

Let us mention briefly other approaches for obtaining initial condition at RHIC and LHC. Commonly used way is to obtain energy density from eq. (8) by using pQCD estimate of the minijet E_T distribution [1, 11, 12, 13]. This formula assumes that the initial partons are uniformly distributed in the coordinate space so that one divides the volume to obtain energy density. In [14] a boost invariant Boltzmann form $f(x,p,t_0) = e^{-p_T \cosh(\eta-y)/T}$ was used for the initial distribution function with $\eta - y$ (boost invariant quantity) correlation taken into account. In the parton cascade model [15] momentum space structure function along with coordinate distribution of the nucleons inside the nucleus was used directly in the transport equation which is different from the minijet calculation by using pQCD. In the approach of this paper we have obtained a phase-space initial non-equilibrium distribution function $f(x,p)$ for the boost non-invariant case with minimal altering the p_T and y distribution of the minijet partons obtained by using pQCD calculations.

To conclude, we have obtained a physically reasonable initial phase-space distribution function of the partons formed at the very early stage of the heavy-ion collisions at RHIC and LHC which can be used to study equilibration of the quark-gluon plasma [13, 14, 16]. To obtain such phase-space distribution function we have considered several plausible parametrizations of $f_i(x,p)$ and have fixed the parameters by matching $\int f(x,p) p^\mu d\sigma_\mu$ to the invariant momentum space semi-hard parton distributions obtained using QCD (pQCD) as well as fitting low order moments of the distribution. Once the parameters of $f(x,p)$ are found, we then have determined the initial number density, energy density and the corresponding (effective) temperature of the minijet plasma at RHIC and LHC energies. For a boost non-invariant minijet phase-space distribution function we obtain $\sim$ 30(140) /fm^3 as the initial number density, $\sim$ 50(520) GeV/fm^3 as the initial energy density and $\sim$ 520(930) MeV as the corresponding initial effective temperature at RHIC(LHC). Our minijet initial conditions are obtained above p_T=1(2) GeV at RHIC(LHC). As partons below these momentum cut-off values might be described by the formation of a classical chromofield, the $f(x, p, t_0)$ obtained in this paper is the distribution function for the matter part. Initial conditions for lower transverse momentum has to be supplied in form of the strength of the chromofield or in the form of field energy density in the non-abelian transport equation to study production and evolution of the quark-gluon plasma at RHIC and LHC.

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[1] X. N. Wang, Phys. Rep. 280 (1997) 287; X. N. Wang and Miklos Gyulassy, Phys. Rev. D 44 (1991) 1991; K. J. Eskola and K. Kajantie, Z. Phys. C 75 (1997) 515; K. J. Eskola, K. Kajantie and J. Lindfors, Nucl. Phys. B 323 (1989) 37; I. Vitev and M. Gyulassy, Phys. Rev. Lett. (in press), hep-ph/0209161 and Talk given by I. Vitev at *hard probes in heavy ion collisions at LHC*, CERN, 7-11 October 2002.

[2] L. M. A. Bettencourt, F. Cooper and K. Pao, Phys. Rev. Lett. 89 (2002) 112301, hep-ph/0109108; L. Mclerran and R. Venugopalan, Phys. Rev. D 49 (1994) 2233, Phys. Rev. D 49 (1994) 3352; Yu. Kovchegov and A. H. Mueller, Nucl. Phys. B 529 (1998) 451; A. Kovner, L. McLerran and H. Weigert, Phys. Rev D 52 (1998) 3809, Phys. Rev. D 52 (1998) 6231.

[3] Y. Kluger, J.M. Eisenberg, B. Svetitsky, F. Cooper and E. Motolla, Phys. Rev. Lett. 67 (1991) 2427; Phys. Rev. D45 (1992) 4659; F. Cooper, J. M. Eisenberg, Y. Kluger, E. Motolla and B. Svetitsky, Phys. Rev. D 48 (1993) 190; C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45 Suppl.1:1-103, 2000.

[4] A. Casher, H. Neuberger and S. Nussinov, Phys. Rev. D20 (1979) 179; D. D. Dietrich, G. C. Nayak and W. Greiner, Phys. Rev. D64 (2001) 074006; hep-ph/0009178; J. Phys. G28 (2002) 2001; G.C. Nayak, D. D. Dietrich and W. Greiner, Rostock 2000/Trento 2001, Exploring Quark Matter 71-78, hep-ph/0104030; G. C. Nayak and V. Ravishankar, Phys. Rev D55 (1997) 6877; Phys. Rev. C58 (1998) 356.

[5] A. H. Mueller, Nucl. Phys. B572 (2000) 227; K. J. Eskola *et al.*, Nucl. Phys. B570 (2000) 379.

[6] M. Glueck, E. Reya and A. Vogt, Euro. Phys. J. C5 (1998) 461.

[7] K. J. Eskola, V. J. Kolhinen and P. V. Ruuskanen, Nucl. Phys. B 535 (1998) 351; K. J. Eskola, V. J. Kolhinen and C. A. Sagado, Euro. Phys. J. C9 (1999) 61.

[8] J.D. Bjorken, Phys. Rev. D**27**, 140 (1983).

[9] E. Shuryak and L. Xiong, Phys. Rev. Lett. 48 (1982) 1066; K. Geiger and J. Kapusta, Phys. Rev. Lett. 70 (1993) 1920; G. C. Nayak, Phys. Lett. B442 (1995) 427; X-M. Xu, D. Kharzeev, H. Satz and X-N. Wang, Phys. Rev. C53 (1996) 3051; G. C. Nayak, JHEP 9802 (1998) 005.

[10] F. Cooper and G. Frye, Phys. Rev. D**10**, 186 (1974).

[11] K. J. Eskola *et al.*, Nucl. Phys. A696 (2001) 715; K. J. Eskola, V. J. Kolhinen, R. Vogt, Nucl. Phys. A696 (2001) 729 and references therein.

[12] K. J. Eskola and M. Gyulassy, Phys. Rev. C47 (1993) 2329.

[13] R.S. Bhalerao, G.C. Nayak, Phys. Rev. C 61 (2000) 054907.

[14] G. C. Nayak, A. Dumitru, L. McLerran and W. Greiner, Nucl. Phys. A 687 (2001) 457.

[15] K. Geiger and B. Muller, Nucl. Phys. B369 (1992) 600; K. Geiger, Phys. Rept. 258 (1995) 237.

[16] C-W. Kao, G. C. Nayak and W. Greiner, Phys. Rev. D66 (2002) 034017; F. Cooper, C-W. Kao and G. C. Nayak, Phys. Rev. D66 (2002) 114016; hep-ph/0207370; J. Ruppert *et al.*, Phys. Lett. B 520 (2002) 233.